Nucleon Exchange in the Composite Particle Scattering

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Abstract : Nucleon exchange in the composite particle scattering is systematically investigated by analyzing the norm kernel. Importance of a few nucleon exchange in the peripheral region of the scattering is shown. The nucleon number exchanged increases as a composite particle system becomes heavier.

§ 1. Introduction

In a composite particle scattering the Pauli principle becomes important. Although the conventional optical potential is useful, such a phenomenon as backward angle anomaly has not been understood in this framework.[1),2)] It seems necessary to study the composite particle scattering in the microscopic theories which take into account the Pauli principle. Recent progress[3)] in the resonating group method and the generator coordinate method has made it possible to treat the composite particle scattering including nuclei heavier than the 0s-clusters. It is laborious, however, to treat a heavier system in the full microscopic theories.

Saito's orthogonality condition model (OCM)[4)], which is approximated from the resonating group method, is a powerful and tractable microscopic model. It has been successfully and widely applied[3),5)] to the study of the composite particle scattering and the cluster structure of light nuclei. The OCM takes into account the Pauli principle in the overlap region by excluding the forbidden states from the model space.

As the composite particle system becomes heavier, it seems necessary to take into consideration further the exchange effect not included in the OCM in the peripheral region. It is interesting to study the exchange effect paying our attention to the nucleon number exchanged in the scattering process.[6),7)] In the present note we systematically investigate the nucleon number exchanged in the composite particle scattering including not only the light nuclei but also the heavier nuclei by analyzing the norm kernel.

In § 2 the eigenvalue problems of the norm kernel are solved and the properties of the eigenvalues are discussed from the viewpoint of the nucleon number exchanged. Concluding remarks are given in § 3.

§ 2. Eigenvalue of the norm kernel and the nucleon number exchanged

We consider first the eigenvalue problem of the norm kernel for the alpha+nucleus system. The norm kernel and the eigenvalue equation are given by

$$\langle h_L | \mathcal{A} \{h_L \chi_L(r)\} \rangle = (1-K_L)\chi_L(r), \tag{1}$$

$$(1-K_L)X_L{}^N(r) = \mu_N X_L{}^N(r) \quad , \tag{2}$$

where $\mathcal{A}$ is the antisymmetrization operator and $h_L=\phi_\alpha\phi_A Y_L(\Omega)$, ϕ_α, ϕ_A being the internal wave functions of the alpha and the target nucleus respectively and assumed to be the harmonic oscillator functions with a common width parameter. The eigenfunction X_L^N is the harmonic oscillator wave function with quanta N and its eigenvalue μ_N is independent of the orbital angular momentum L in the cases of the closed shell nuclei.

From the viewpoint of the nucleon number exchanged, the eigenvalue μ_N can be written as

$$\mu_N = \sum_{n=0}^{4} \mu_N^{(n)} . \tag{3}$$

Here n means the nucleon number exchanged between two composite particle nuclei and $\mu_N^{(n)}$ is the eigenvalue of the norm kernel corresponding to the n nucleon exchange. The eigenfunction is also the harmonic oscillator wave function.[6]

The eigenvalue problems for the closed shell target nuclei of ^{16}O, ^{40}Ca and ^{80}Zr are solved and the eigenvalues for n nucleon exchange are given as follows:

for $\alpha+{}^{16}$O

$$\mu_N^{(n)} = (-)^n \binom{4}{n} \sum_{r=0}^{n} \binom{n}{r} \left(\frac{5}{16}\right)^r \left(1-\frac{5}{16}n\right)^{N-r} \frac{N!}{(N-r)!}, \tag{4}$$

for $\alpha+{}^{40}$Ca

$$\mu_N^{(n)} = (-)^n \binom{4}{n} \sum_{r=0}^{n} \binom{n}{r} \sum_{s=0}^{r} \binom{r}{s} \left(\frac{11}{40}\right)^{2r-s} \left(1-\frac{11}{40}n\right)^{N+s-2r} \frac{1}{2^r} \frac{N!}{(N+s-2r)!}, \tag{5}$$

and for $\alpha+{}^{80}$Zr

$$\mu_N^{(n)} = (-)^n \binom{4}{n} \sum_{r=0}^{n} \binom{n}{r} \sum_{s=0}^{r} \binom{r}{s} \sum_{t=0}^{s} \binom{s}{t} \left(\frac{21}{80}\right)^{r+s+t} \left(1-\frac{21}{80}n\right)^{N-r-s-t} \cdot \frac{N!}{(N-r-s-t)!}. \tag{6}$$

Here ^{80}Zr is a closed shell nucleus in the sense it fully occupies the oscillator state of quantum number $N=3$.

The calculated eigenvalues of the norm kernel are shown in Fig. 1 together with the $\alpha+\alpha$ system. For the $\alpha+\alpha$ case the eigenvalue of the first allowed state is very large having $\mu=0.75$. As the mass of the target nucleus increases the eigenvalues of the first allowed states become rapidly small. This corresponds to a degree of development of the alpha cluster structure in nuclei as follows: The ^{8}Be nucleus has a typical well developed cluster structure,

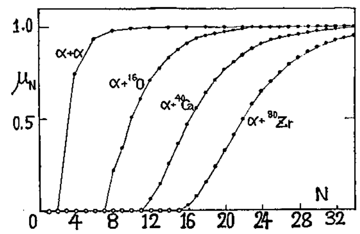

Fig. 1. The eigenvalues μ_N of the norm kernel for $\alpha+\alpha$, $\alpha+{}^{16}$O, $\alpha+{}^{40}$Ca and $\alpha+{}^{80}$Zr. The forbidden states are indicated by white circles.

while ^{20}Ne has not a well developed $\alpha+^{16}$O cluster structure but has a transient character between molecule-like structure and shell structure. For ^{44}Ti it is considered to have a shell structure in the $N=12$ quantum state. If a cluster structure could arise, it would be expected at the high excited energy region which correspnds to the second or the third allowed states. In the mass region of $A=80$ it will be difficult to expect a cluster aspect in the first allowed state whose eigenvalue is less than 0.1. This state can be considered as an almost forbidden state.

Next we investigate the eigenvalues of the norm kernel deviding into each nucleon exchange. In Table I the calculated eigenvalues of $\alpha+^{40}$Ca and $\alpha+^{80}$Zr for each nucleon number exchanged are given. It is clearly seen that one nucleon exchange is dominant in the allowed states and the eigenvalues decrease rapidly as the nucleon number exchanged increases. Although in the $\alpha+^{16}$O system the four nucleon exchange part is greater than the three nucleon exchange one, in the present $\alpha+^{40}$Ca and $\alpha+^{80}$Zr systems the former is smaller than the latter. Therefore the situation seen in the $\alpha+^{16}$O system is not in general. The eigenvalues of the three and the four nucleon exchange are some orders of magnitude smaller than the one nucleon exchange.

In the $\alpha+^{40}$Ca case the total eigenvalue μ_N is closely approximated by the sum of the zero, one and two nucleon exchanges in the $N=12\sim16$, and for the $N=16\sim30$ only one nucleon exchange is dominant. If the oscillator width parameter is taken as $\nu=0.14$ ($\hbar\omega=\frac{\hbar^2}{m}2\nu$), the effective range of the harmonic oscillator wave function of $N=12$ roughly corresponds to 3.64 fm and $N=16$ to 4.15 fm, according to $\sqrt{\langle X_L^N|r^2|X_L^N\rangle}=\sqrt{(N+3/2)/2\gamma}$ ($\gamma=\frac{4A}{A+4}\nu$).

When the two nuclei of α and ^{40}Ca are coming near first one nucleon exchange effect appears around the contact region. Coming near further each other two nucleon exchange effect also arises. In the effective range of the allowed states more than three nucleon exchange effect is negligible. This situation also holds for the $\alpha+^{80}$Zr and $\alpha+^{16}$O systems.

We proceed to the further heavier systems including the ^{16}O nucleus as a projectile and study the ^{16}O$+^{40}$Ca and ^{16}O$+^{80}$Zr systems. The eigenvalues of the norm kernel are expressed as follows:

$$\mu_N=\sum_{n=0}^{16}\mu_N^{(n)}\ , \tag{7}$$

and for ^{16}O$+^{40}$Ca

$$\mu_N^{(n)}=(-)^n\sum_{k=0}^{4}\binom{4}{k}\frac{1}{2^k}\sum_{r=0}^{k}(-)^r\binom{k}{r}\sum_{s=k}^{k+r}(-)^s\binom{r}{s-k}\binom{16-2k}{n-s}\cdot\sum_{t=0}^{n-s}\binom{n-s}{t}\left(\frac{7}{80}\right)^{3k+r+t-s}\left(1-\frac{7}{80}n\right)^{N-3k-r-t+s}\frac{N!}{(N-3k-r-t+s)!}\ , \tag{8}$$

and for ^{16}O$+^{80}$Zr

$$\mu_N^{(n)}=(-)^n\sum_{k=0}^{4}(-)^k\binom{4}{k}\sum_{l=0}^{4-k}\binom{4-k}{l}\sum_{p=0}^{8}\binom{8}{p}\binom{4-k}{n-2k-l-p}\sum_{q=0}^{p}\binom{p}{q}$$

Table I. The eigenvalues $\mu_N^{(n)}$ for $\alpha+{}^{40}Ca$ and $\alpha+{}^{80}Zr$. The total eigenvalues μ_N are also

(a) $\alpha+{}^{40}Ca$

	N	0	2	4	6	8	10	12	14
even parity	$\mu_N^{(1)}$	−4.0	−4.0	−3.736	−3.157	−2.462	−1.808	−1.269	−0.860
	$\mu_N^{(2)}$	6.0	6.0	5.208	3.521	1.895	0.859	0.343	0.125
	$\mu_N^{(3)}$	−4.0	−4.0	−3.208	−1.573	−0.405	-6.29^{2}	-7.00^{3}	-6.23^{4}
	$\mu_N^{(4)}$	1.0	1.0	0.736	0.208	-2.83^{2}	1.21^{2}	2.21^{3}	1.62^{4}
	μ_N	0.0	0.0	0.0	0.0	0.0	0.0	6.91^{2}	0.264
	N	1	3	5	7	9	11	13	15
odd parity	$\mu_N^{(1)}$	−4.0	−3.917	−3.474	−2.812	−2.124	−0.152	−1.049	−0.701
	$\mu_N^{(2)}$	6.0	5.750	4.421	2.649	1.300	0.550	0.209	7.32^{2}
	$\mu_N^{(3)}$	−4.0	−3.750	−2.421	−0.862	−0.168	-2.17^{2}	-2.14^{3}	-1.75^{4}
	$\mu_N^{(4)}$	1.0	0.917	0.474	2.52^{2}	-8.08^{3}	-6.19^{2}	-6.42^{4}	-3.66^{5}
	μ_N	0.0	0.0	0.0	0.0	0.0	0.0	0.157	0.372

(b) $\alpha+{}^{80}Zr$

	N	0	2	4	6	8	10	12	14
even parity	$\mu_N^{(1)}$	−4.0	−4.0	−3.981	−3.822	−3.473	−2.984	−2.439	−1.911
	$\mu_N^{(2)}$	6.0	6.0	5.943	5.465	4.427	3.097	1.885	1.018
	$\mu_N^{(3)}$	−4.0	−4.0	−3.943	−3.465	−2.437	−1.242	−0.437	−0.111
	$\mu_N^{(4)}$	1.0	1.0	0.981	0.822	0.482	0.129	-8.46^{3}	3.94^{3}
	μ_N	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	N	1	3	5	7	9	11	13	15
odd parity	$\mu_N^{(1)}$	−4.0	−4.0	−3.925	−3.669	−3.241	−2.713	−2.170	−1.668
	$\mu_N^{(2)}$	6.0	6.0	5.775	5.008	3.771	2.455	1.404	0.720
	$\mu_N^{(3)}$	−4.0	−4.0	−3.775	−3.008	−1.821	−0.771	−0.229	-5.07^{2}
	$\mu_N^{(4)}$	1.0	1.0	0.925	0.669	0.290	2.85^{2}	-6.18^{3}	-1.24^{3}
	μ_N	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

$$\cdot\sum_{r=0}^{n-2k-l-p}\binom{n-2k-l-p}{r}\sum_{s=0}^{l}\binom{l}{s}\left(\frac{1}{6}\right)^{n-p-r-s}\sum_{t=0}^{s}\binom{s}{t}\left(-\frac{1}{2}\right)^{s-t}$$

$$\cdot\sum_{u=0}^{r+t}\binom{r+t}{u}\sum_{v=0}^{q+u}\binom{q+u}{v}\left(\frac{1}{2}\right)^{v}\left(\frac{3}{40}\right)^{\alpha}\left(1-\frac{3}{40}n\right)^{N-\alpha}\frac{N!}{(N-\alpha)!}, \qquad (9)$$

shown. A small figure denotes minus power of 10, for example $-0.246^{2}=-0.246\times 10^{-2}$.

16	18	20	22	24	26	28	30	32	34
-0.567	-0.366	-0.231	-0.144	-8.86^{2}	-5.39^{2}	-3.24^{2}	-1.94^{2}	-1.15^{2}	-6.75^{3}
4.22^{2}	1.35^{2}	4.12^{3}	1.21^{3}	3.45^{4}	9.58^{5}	2.60^{5}	6.91^{6}	1.81^{6}	4.65^{7}
-4.73^{5}	-3.19^{6}	-1.96^{7}	-1.12^{8}	-6.06^{10}	-3.12^{11}	-1.54^{12}	-7.33^{14}	-3.39^{15}	-1.52^{16}
7.64^{6}	2.76^{7}	8.29^{9}	2.17^{10}	5.10^{12}	1.10^{13}	2.23^{15}	4.25^{17}	7.71^{19}	1.34^{20}
0.475	0.648	0.773	0.857	0.912	0.946	0.968	0.981	0.989	0.993

17	19	21	23	25	27	29	31	33	35
-0.456	-0.291	-0.183	-0.113	-6.92^{2}	-4.19^{2}	-2.51^{2}	-1.49^{2}	-8.81^{3}	-5.17^{3}
2.40^{2}	7.50^{3}	2.24^{3}	6.49^{4}	1.82^{4}	5.00^{5}	1.34^{5}	3.54^{6}	9.18^{7}	2.35^{7}
-1.24^{5}	-7.99^{7}	-4.73^{8}	-2.63^{9}	-1.34^{10}	-6.96^{12}	-3.37^{13}	-1.58^{14}	-7.21^{16}	-3.21^{17}
-1.49^{6}	-4.88^{8}	-1.36^{9}	-3.37^{11}	-7.58^{13}	-1.58	-3.10^{16}	-5.76^{18}	-1.02^{19}	-1.75^{21}
0.568	0.716	0.819	0.887	0.931	0.958	0.975	0.985	0.991	0.995

16	18	20	22	24	26	28	30	32	34
-1.445	-1.060	-0.758	-0.530	-0.364	-0.246	-0.164	-0.108	-7.01^{2}	-4.52^{2}
0.498	0.225	9.47^{2}	3.78^{2}	1.44^{2}	5.24^{3}	1.85^{3}	6.32^{4}	2.11^{4}	6.85^{5}
-2.19^{2}	-3.53^{3}	-4.90^{4}	-6.02^{5}	-6.69^{6}	-6.87^{7}	-6.58^{8}	-5.95^{9}	-5.13^{10}	-4.23^{11}
2.82^{4}	8.26^{6}	1.51^{7}	2.03^{9}	2.19^{11}	2.00^{13}	1.61^{15}	1.16^{17}	7.64^{20}	4.68^{22}
3.21^{2}	0.162	0.337	0.508	0.650	0.759	0.838	0.893	0.930	0.955

17	19	21	23	25	27	29	31	33	35
-1.241	-0.899	-0.635	-0.440	-0.300	-0.201	-0.133	-8.71^{2}	-5.64^{2}	-3.61^{2}
0.338	0.147	6.02^{2}	2.34^{2}	8.71^{3}	3.12^{3}	1.08^{3}	3.66^{4}	1.20^{4}	3.88^{5}
-8.98^{3}	-1.34^{3}	-1.74^{4}	-2.03^{5}	-2.16^{6}	-2.14^{7}	-1.99^{8}	-1.76^{9}	-1.48^{10}	-1.20^{11}
-5.20^{5}	-1.17^{6}	-1.81^{8}	-2.16^{10}	-2.13^{12}	-1.82^{14}	-1.38^{16}	-9.51^{19}	-6.03^{21}	-3.58^{23}
8.72^{2}	0.247	0.425	0.583	0.709	0.802	0.868	0.913	0.944	0.964

where $\alpha=3n+k+l-3p+q-3r-s-3t+u+v$.

The first allowed states are $N=24$ for $^{16}O+^{16}O$, $N=40$ for $^{16}O+^{40}Ca$ and $N=56$ for $^{16}O+^{80}Zr$. The calculated total eigenvalues for $^{16}O+^{40}Ca$ are shown in Fig. 2 in comparison with the $^{16}O+^{16}O$ case. It is clearly seen that the number of the almost forbidden states increases as the composite particle system becomes heavier. The quantum number where μ_N becomes one half is very large in comparison with the alpha+nucleus system; $N=42$ for $^{16}O+^{16}O$ and $N=80$ for $^{16}O+^{40}Ca$. For the $^{16}O+^{80}Zr$ case the eigenvalues of the almost

Table II. The eigenvalues of $\mu_N^{(n)}$ for $^{16}O+^{40}Ca$. Only the eigenvalues of even

N	0	2	4	6	8	10	12	14	16	18	20	22	24
$\mu_N^{(1)}$	−16.0	−15.85	−15.22	−14.36	−13.38	−12.39	−11.44	−10.54	−9.72	−8.97	−8.29	−7.68	−7.12
$\mu_N^{(2)}$	120.0	117.7	108.4	95.98	82.92	70.63	59.75	50.43	42.58	36.02	30.56	26.00	22.17
$\mu_N^{(3)}$	−560.0	−543.9	−479.4	−396.5	−315.2	−245.1	−188.6	−144.5	−110.9	−85.28	−65.83	−50.98	−39.58
$\mu_N^{(4)}$	1820.0	1750.3	1472.7	1131.2	821.0	577.3	400.0	275.8	190.3	131.8	91.54	63.79	44.51
$\mu_N^{(5)}$	−4368.0	−4159.0	−3331.9	−2360.3	−1547.9	−973.2	−600.2	−368.0	−225.7	−138.9	−85.66	−52.90	−32.63
$\mu_N^{(6)}$	8008.0	7548.2	5741.2	3719.3	2176.1	1205.9	652.6	350.6	188.3	101.2	54.44	29.19	15.55
$\mu_N^{(7)}$	−11440.0	−10674.	−7682.8	−4504.3	−2312.7	−1109.1	−517.1	−238.9	−110.0	−50.57	−23.11	−10.45	−4.65
$\mu_N^{(8)}$	12870.0	11885.	8066.2	4223.0	1861.0	754.0	295.6	114.4	43.96	16.71	6.25	2.28	0.805
$\mu_N^{(9)}$	−11440.0	−10455.	−6663.1	−3061.1	−1121.9	−371.5	−118.5	−37.03	−11.36	−3.40	−0.977	−0.266	-6.79^{2}
$\mu_N^{(10)}$	8008.0	7241.6	4313.5	1697.7	493.1	127.4	31.38	7.47	1.70	0.363	7.08^{2}	1.23^{2}	1.82^{3}
$\mu_N^{(11)}$	−4368.0	−3908.2	−2163.8	−704.8	−149.1	−28.14	−4.85	−0.792	−0.112	-1.32^{2}	-1.22^{3}	-6.81^{5}	-2.11^{6}
$\mu_N^{(12)}$	1820.0	1611.0	823.8	210.3	26.92	3.54	0.277	3.18^{2}	2.39^{3}	-1.04^{3}	5.03^{5}	5.21^{5}	6.00^{6}
$\mu_N^{(13)}$	−560.0	−490.3	−229.8	−41.73	−1.46	−0.305	3.90^{2}	-1.66^{3}	-3.87^{3}	1.81^{3}	6.51^{5}	-2.35^{4}	-2.78^{5}
$\mu_N^{(14)}$	120.0	103.9	44.25	4.58	−0.403	7.74^{2}	-1.37^{2}	-2.09^{3}	3.75^{3}	-1.54^{3}	-2.18^{4}	3.53^{4}	5.42^{5}
$\mu_N^{(15)}$	−16.0	−13.70	−5.24	−0.105	6.41^{2}	-1.48^{2}	1.44^{3}	1.77^{3}	-1.71^{3}	6.46^{4}	1.74^{4}	-2.29^{4}	-4.52^{5}
$\mu_N^{(16)}$	1.0	0.847	0.287	-2.06^{2}	-2.45^{4}	3.12^{4}	2.39^{4}	-3.97^{4}	3.06^{4}	-1.08^{4}	-4.58^{5}	5.52^{5}	1.35^{5}
μ_N	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

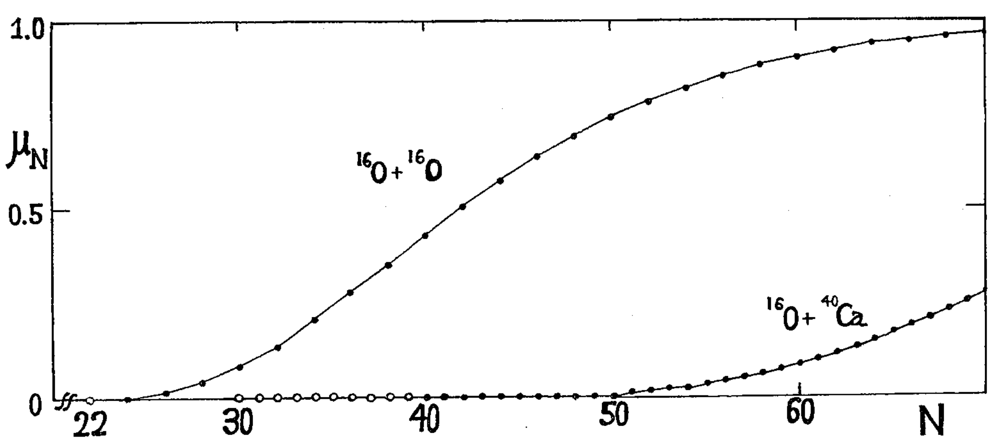

Fig. 2. The eigenvalues μ_N of the norm kernel of $^{16}O+^{16}O$ and $^{16}O+^{40}Ca$.
The forbidden states are indicated by white circles.

forbidden states become vanishingly small. Some of the results are shown in Table II.

The eigenvalues for the each nucleon exchange of the $^{16}O+^{40}Ca$ system are calculated and shown in Table III. As the two nuclei of ^{16}O and ^{40}Ca come near the number of nucleons exchanged increases. Around the effective range of N~60 it is sufficient to conparity are shown. See also the caption of Table I.

26	28	30	32	34	36	38	40	42	44	50	56	62	68
−6.61	−6.14	−5.71	−5.30	−4.93	−4.58	−4.24	−3.93	−3.64	−3.36	−2.62	−2.01	−1.51	−1.12
18.94	16.19	13.85	11.84	10.10	8.61	7.32	6.20	5.24	4.41	2 57	1.45	0.785	0.411
−30.77	−23.93	−18.59	−14.41	−11.13	−8.57	−6.57	−5.00	−3.79	−2.86	−1.18	−0.457	−0 168	-5.84^{2}
31.06	21.62	15.00	10.36	7.10	4.83	3.26	2.18	1.45	0.951	0.254	6.23^{2}	1.41^{2}	2.97^{3}
−20.05	−12.24	−7.41	−4.44	−2.63	−1.54	−0.886	−0.503	−0.281	−0.155	-2.36^{2}	-3.16^{3}	-3.77^{4}	-4.05^{5}
8.20	4.27	2.18	1.10	0.538	0.259	0.121	5.56^{2}	2.49^{2}	1.09^{2}	7.97^{4}	4.87^{5}	2.54^{6}	1.16^{7}
−2.03	−0.860	−0.354	−0.141	-5.41^{2}	-2.00^{2}	-7.16^{3}	-2.47^{3}	-8.22^{4}	-2.65^{4}	$-7\ 40^{6}$	-1.63^{7}	-2.97^{9}	-4.60^{11}
0.273	8.79^{2}	2.69^{2}	7.76^{3}	2.12^{3}	5.47^{4}	1.34^{4}	3.12^{5}	6.93^{6}	1.48^{6}	1.13^{8}	6.52^{11}	2.99^{13}	1.14^{15}
-1.60^{2}	-3.43^{3}	-6.74^{4}	-1.19^{4}	-1.96^{5}	-2.95^{6}	-4.15^{7}	-5.46^{8}	-6.76^{9}	-7.94^{10}	-9.73^{13}	-8.53^{16}	-5.75^{19}	-3.16^{22}
2.27^{4}	2.37^{5}	2.11^{6}	1.63^{7}	1.11^{8}	6.78^{10}	3.77^{11}	1.93^{12}	9.18^{14}	4.09^{15}	2.63^{19}	1.15^{23}	3.75^{28}	9.73^{33}
-4.16^{8}	-5.80^{10}	-6.23^{12}	-5.44^{14}	-4.02^{16}	-2.58^{18}	-1.48^{20}	-7.67^{23}	-3.65^{25}	-1.61^{27}	-9.54^{35}	-3.68^{42}	$-1\ 03^{49}$	-2.22^{57}
3.27^{7}	1.12^{8}	2.73^{10}	5.19^{12}	8.06^{14}	1.07^{15}	1.23^{17}	1.27^{19}	1.19^{21}	1.03^{23}	4.35^{30}	1.14^{36}	2.10^{43}	2.95^{50}
1.44^{5}	6.16^{6}	1.30^{6}	1.92^{7}	2.23^{8}	2.16^{9}	1.82^{10}	1.36^{11}	9.25^{13}	5.76^{14}	9.27^{18}	9.35^{22}	6.69^{26}	3.67^{30}
-4.61^{5}	-1.93^{5}	-1.46^{7}	2.55^{6}	1.30^{6}	4.23^{7}	1.07^{7}	2.30^{8}	4.34^{9}	7.37^{10}	2.27^{12}	4.22^{15}	5.47^{18}	5.42^{21}
4.68^{5}	2.04^{5}	-3.65^{6}	-5.78^{6}	-1.95^{6}	1.97^{7}	5.78^{7}	3.82^{7}	1.77^{7}	6.74^{8}	1.83^{9}	2.61^{11}	2.45^{13}	1.72^{15}
-1.56^{5}	-7.18^{6}	2.48^{6}	3.05^{6}	6.21^{7}	-6.32^{7}	-5.99^{7}	-2.09^{7}	4.90^{8}	1.25^{7}	3.56^{8}	2.87^{9}	1.32^{10}	4.24^{12}
0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.550^{6}	5.528^{5}	3.228^{4}	7.408^{3}	4.117^{2}	0.1185	0.2356

sider at least three nucleon exchange. If they close together further the number of nucleons exchanged also increases and in the range of $N\sim40$, the first allowed state, five nucleon exchange effect arises. For the $^{16}O+^{80}Zr$ system it is at least necessary to consider six nucleon exchange effect in the $N\sim56$ region. While in the case of the $^{16}O+^{16}O$ system it is sufficient to take account of three nucleon exchange in the $N\sim24$ region where the first allowed state occurs. In is found that the number of nucleons exchanged in the first

Table III. The eigenvalues μ_N of the almost forbidden states for $^{16}O+^{80}Zr$. Only the eigenvalues of even parity are shown.

N	μ_N	N	μ_N
56	0.6202×10^{-8}	72	0.2739×10^{-2}
58	0.2597×10^{-6}	74	0.5112×10^{-2}
60	0.2804×10^{-5}	76	0.8830×10^{-2}
62	0.1663×10^{-4}	78	0.1429×10^{-1}
64	0.6846×10^{-4}	80	0.2188×10^{-1}
66	0.2190×10^{-3}	82	0.3194×10^{-1}
68	0.5814×10^{-3}	84	0.4476×10^{-1}
70	0.1336×10^{-2}	86	0.6051×10^{-1}

allowed state region increases as the composite particle system becomes heavier. It is worth noting that this is related with the appearance of the almost forbidden states.

We have seen in all the systems considerd one nucleon exchange effect is important. It is interesting if we can know the nucleon exchange between the shell orbits of the nuclei. For simplicity we consider the $\alpha+{}^{16}O$ system. The eigenvalue of n nucleon exchange is rewritten as

$$\mu_N{}^{(n)}=(-)^n\binom{4}{n}\sum_{r=0}^{n}\binom{n}{r}\left(-\frac{5}{16}\right)^r\left(1-\frac{5}{16}n\right)^{N-r}\frac{N!}{(N-r)!}\varepsilon_1{}^{n-r}\varepsilon_2{}^r, \qquad (10)$$

where $\varepsilon_1=\varepsilon_2=1$ and $n-r$, r mean the nucleon number exchanged from the $0p$ orbit and the $0s$ orbit of the ^{16}O nucleus respectively. The calculated values are given in Table IV. In the one particle exchange process the exchange from the nucleon in the $0p$ shell orbit is

Table IV. The eigenvalues $\mu_N{}^{(n)}$ for $\alpha+{}^{16}O$ are shown divided into each shell orbits. n_1, n_2 mean the nucleon number exchanged from the $0s$, $0p$ orbits of the ^{16}O nucleus respectively.

n	n_1 ($0s$)	n_2 ($0p$)	$N=8$	9	10	11
1	0	1	−0.726	−0.561	−0.429	−0.324
	1	0	−0.200	−0.137	-0.944^{1}	-0.649^{1}
2	0	2	0.912^{1}	0.440^{1}	0.206^{1}	0.945^{2}
	1	1	0.313^{1}	0.132^{1}	0.550^{2}	0.227^{2}
	2	0	0.235^{2}	0.880^{3}	0.330^{3}	0.124^{3}
3	0	3	-0.391^{4}	-0.367^{5}	-0.327^{6}	-0.281^{7}
	1	2	-0.391^{5}	-0.314^{6}	-0.246^{7}	-0.188^{8}
	2	1	-0.112^{7}	-0.786^{8}	-0.546^{9}	-0.375^{10}
	3	0	-0.931^{9}	-0.582^{10}	-0.364^{11}	-0.227^{12}
4	0	4	0.621^{1}	-0.282^{1}	0.117^{1}	-0.461^{2}
	1	3	-0.401^{1}	0.150^{1}	-0.536^{2}	-0.184^{2}
	2	2	0.801^{2}	-0.257^{2}	0.805^{3}	-0.246^{3}
	3	1	-0.610^{3}	0.172^{3}	-0.477^{4}	0.131^{4}
	4	0	0.153^{4}	-0.381^{5}	0.954^{6}	-0.238^{6}
μ_N			0.229	0.344	0.510	0.620

greater than that in the $0s$ orbit. For any number of nucleons exchanged in the allowed state the exchange process including the nucleons in the $0p$ orbit is the greatest of all. The same situation is seen in the $\alpha+{}^{40}Ca$ system and so on. It is therefore desirable to firstly consider the one nucleon exchange from the outermost shell orbit in the composite particle scattering.

§ 3. Concluding remarks

We have studied the composite particle scattering from the viewpoint of nucleon number exchanged by analyzing the norm kernel. It is found that the almost forbidden states increase as the composite particle becomes heavier and the eigenvalues become rapidly small. In the $^{16}O+^{40}Ca$ and $^{16}O+^{80}Zr$ cases the first eigenvalues are in the magnitude of order minus five and minus eight respectively. It is also pointed out that the one nucleon exchange effect is most dominant. As the composite particle becomes heavier nucleon number exchanged which makes a significant contribution to the norm kernel increases. By analyzing the nucleon exchange between shell orbits of the nuclei it is found that the exchange from the outermost shell orbit is most important. By taking the exchange from the outermost shell orbit, the OCM will be improved. It is interesting to study further the composite particle scattering including heavier nuclei systematically with a dynamical treatment of the Pauli principle[8] from the viewpoint of nucleon number exchanged.

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